

Short Papers

A Novel Waveguide Y-Junction Circulator with a Ferrite Sphere for Millimeter Waves

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Abstract—A novel waveguide Y-junction circulator with a ferrite sphere is developed for uses at millimeter-wave frequencies. Sphere is preferred because the production cost of a pertinent circulator is lower. Performance of the new circulator is comparable to the conventional one with a cylindrical ferrite. A simple formula is derived for determining the center frequency of a circulator. Good agreement between analytical results and measurements is observed. An extremely fast method is developed to analyze the circulator. Various properties of the circulator can be predicted with reasonable accuracy.

I. INTRODUCTION

H-plane waveguide Y-junction circulators are extensively used in microwave systems. The most popular anisotropic material used to guide the electromagnetic wave to circulate is a ferrite post of full or partial height [1]. In addition to circular ferrite cylinders, triangular or hexagonal posts can also be used [2], [3]. For uses at millimeter-wave frequencies, a similar Y-junction circulator can be constructed but it is proportionally smaller in size. Observe that the relative permittivity of ferrite is very high ($\epsilon_r > 10$); therefore, the diameter of a mm-wave ferrite cylinder is of the order of 1 mm. Due to hardness of ferrite and smallness of the cylinder, ultra-high-precision machinery is needed to produce a cylinder according to a prescribed specification. Then adept craftsmanship is needed to mount the diminutive cylinder normally to the floor inside the equally tiny waveguide junction. To address both of the aforementioned problems, an innovative idea has been contemplated. In essence, it is to replace the ferrite cylinder by a ferrite sphere. The latter is chosen because ferrite spheres can be made as if they were ball bearings. As tiny ball bearings of diameter of 1 mm or less are widely used in micro-machines for decades, relevant production techniques are very mature. Furthermore due to spherical symmetry, a sphere is always "normal" to the floor of the waveguide junction.

II. CONFIGURATION OF THE CIRCULATOR

The configuration of an H-plane waveguide Y-junction circulator with a ferrite sphere of radius R is sketched in Fig. 1. The circulator is designed for uses in the 4-mm-wave range. It is fed by three WR12 waveguides of width 3.099 mm and of height 1.549 mm. TE_{10} mode is the only mode of propagation in the waveguide. Gyrotropic Al-Ni-Zn ferrite spheres of relative permittivity $\epsilon_r = 13.5$, of loss tangent $\sigma = 0.0005$, and of saturated magnetization $4\pi M_s = 4950$ Gauss are selected. The radius of the ferrite sphere varies from 0.5–0.6 mm. It is fixed on top of a 0.2-mm-thick triangular impedance matching pedestal inside the waveguide junction. With the thickness of epoxy glue included, the sphere is approximately placed at the center of the waveguide junction. For easy reference, the circulator is placed on the x - y plane. While port 1 at $\phi = 0$ is designated as the input

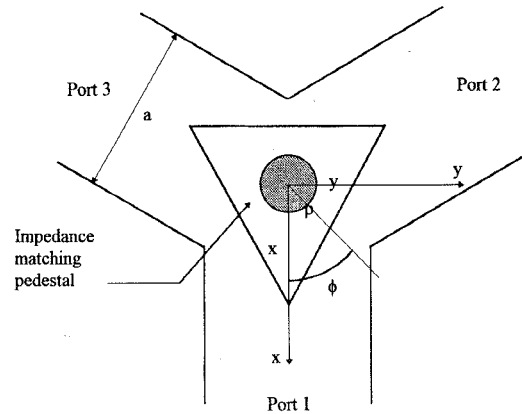


Fig. 1. Configuration of an H-plane waveguide Y-junction circulator with a ferrite sphere.

channel; port 2 ($\phi = 2\pi/3$) and port 3 ($\phi = 4\pi/3$) are output channels. The sphere is saturately magnetized in the z -direction by two electromagnets above and below the junction.

III. PERMEABILITY TENSOR OF A MAGNETIZED FERRITE SPHERE

For a gyrotropic ferrite sample magnetized in the z -direction, the permeability tensor $[\mu_r]$ in the Cartesian coordinate system is [4]

$$[\mu_r] = \mu_0 \begin{bmatrix} \mu & j\kappa & 0 \\ -j\kappa & \mu & 0 \\ 0 & 0 & 1 \end{bmatrix}. \quad (1)$$

Elements in the above tensor are

$$\mu = 1 + \frac{\omega_0 \omega_m}{\omega_0^2 - \omega^2} \quad (2)$$

$$\kappa = \frac{\omega \omega_m}{\omega_0^2 - \omega^2} \quad (3)$$

where $\omega_0 = \mu_0 \gamma H_0$ is the precession frequency and $\omega_m = \mu_0 \gamma M_s$. While H_0 is the bias magnetic field; $\gamma = 1.759 \times 10^7$ Hz/Gauss is the gyromagnetic ratio.

The permeability tensor $[\mu_s]$ in spherical coordinates can be obtained by the following transformation

$$[\mu_s] = [T][\mu_r][T]^{-1}. \quad (4)$$

The transformation matrix is

$$[T] = \begin{bmatrix} \sin \theta \cos \phi & \sin \theta \sin \phi & \cos \theta \\ \cos \theta \cos \phi & \cos \theta \sin \phi & -\sin \theta \\ -\sin \phi & \cos \phi & 0 \end{bmatrix}. \quad (5)$$

For easy reference, $[\mu_s]$ is explicitly given below

$$[\mu_s] = \mu_0 \begin{bmatrix} \mu \sin^2 \theta + \cos^2 \theta & (\mu - 1) \sin \theta \cos \theta & j\kappa \sin \theta \\ (\mu - 1) \sin \theta \cos \theta & \mu \cos^2 \theta + \sin^2 \theta & j\kappa \cos \theta \\ -j\kappa \sin \theta & -j\kappa \cos \theta & \mu \end{bmatrix}. \quad (6)$$

It is interesting to note that $[\mu_s]$ is independent of ϕ .

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IV. CENTER FREQUENCY OF THE CIRCULATOR

Traditionally, the frequency range of a circulator and its center frequency are determined by solving the tensor Maxwell's equation in a ferrite specimen of a well-defined shape under a given magnetic bias. Since the permeability tensor in spherical coordinates is much more complex than its counterpart in Cartesian or cylindrical coordinate system, the equations involved are proportionally more complex. It is doubtful whether the equations can be solved in a closed form. To this end, a new method is developed to solve the problem. In this section, a simple formula is derived to determine the center frequency of a circulator. An extremely fast technique will then be developed to approximately find other properties of the circulator in subsequent sections.

For a waveguide Y-junction circulator with a ferrite cylinder, Owen [5] showed that the center frequency is approximately equal to the resonant frequency of a ferrite cylinder as if the latter were a dielectric resonator. Along this line of thought, the ferrite sphere is first examined as a dielectric resonator with no external magnetic bias. Since the relative permittivity of ferrite is much larger than unity, the sphere is assumed to be completely shielded by a magnetic wall. The electromagnetic fields inside the sphere are expressed in terms of transverse-electric and transverse-magnetic waves, relative to the radial direction. As the ferrite sphere is placed inside an H-plane waveguide junction circulator, the $TE_{mn\ell}$ modes of interest are the modes with the following components

$$E_r = 0, \quad (7a)$$

$$E_\theta = j \frac{m\beta\eta}{r \sin \theta} \tilde{J}_n(\beta r) P_n^m(\cos \theta) \left\{ \begin{array}{c} \sin \\ -\cos \end{array} \right\} m\phi, \quad (7b)$$

$$E_\phi = j \frac{\beta\eta}{r} \tilde{J}_n(\beta r) \frac{dP_n^m(\cos \theta)}{d\theta} \left\{ \begin{array}{c} \cos \\ \sin \end{array} \right\} m\phi, \quad (7c)$$

$$H_r = \frac{n(n+1)\eta}{r^2} \tilde{J}_n(\beta r) P_n^m(\cos \theta) \left\{ \begin{array}{c} \cos \\ \sin \end{array} \right\} m\phi, \quad (7d)$$

$$H_\theta = \frac{1}{r} \frac{d\tilde{J}_n(\beta r)}{dr} \frac{dP_n^m(\cos \theta)}{d\theta} \left\{ \begin{array}{c} \cos \\ \sin \end{array} \right\} m\phi, \quad (7e)$$

$$H_\phi = \frac{m}{r \sin \theta} \frac{d\tilde{J}_n(\beta r)}{dr} P_n^m(\cos \theta) \left\{ \begin{array}{c} -\sin \\ \cos \end{array} \right\} m\phi, \quad (7f)$$

$m = 0, 1, \dots, n; \quad n = 1, 2, \dots;$

where β is the propagation constant to be determined, $\eta = 120\pi/\sqrt{\epsilon_r}$ is the intrinsic impedance, and P_n^m is an associated Legendre polynomial. \tilde{J}_n is defined as

$$\tilde{J}_n(u) = u j_n(u) = \sqrt{\frac{\pi u}{2}} J_{n+1/2}(u) \quad (8)$$

where j_n is a spherical Bessel's function of the first kind. The characteristic equation of a $TE_{mn\ell}$ wave in the ferrite sphere is

$$\tilde{J}'_n(\beta R) = 0. \quad (9)$$

Admissible propagation constants $\beta_{nm\ell}$ are obtained by solving (9). Although the lowest mode of operation in the dielectric resonator is TE_{101} , the lowest mode in the junction circulator is TE_{111} because the former does not provide a ϕ -dependent wave. The smallest root is $\beta_{111}R = 2.744$. Thus, the center frequency of a circulator f_0 is

$$f_0 = \frac{1.372c}{\pi R \sqrt{\epsilon_r}}. \quad (10)$$

V. ELECTROMAGNETIC FIELD IN A MAGNETIZED FERRITE SPHERE

When a ferrite sphere is being magnetized, the field inside will undergo some changes. No major changes are expected because the boundary condition remains intact. Although a single $TE_{mn\ell}$ mode could not satisfy the tensor Maxwell's equations, a series of $TE_{mn\ell}$ modes could. The innovation is to search for a minimum combination of eigenmodes that could approximately satisfy the wave equations. Assume that E_θ and E_ϕ can be adequately represented by the lowest two eigenmodes only. By combining the sine and cosine terms, E_θ and E_ϕ are

$$E_\theta = -\frac{\beta\eta}{r} \left[\frac{\sin(\beta r)}{\beta r} - \cos(\beta r) \right] \cos(\phi + \phi_0), \quad (11a)$$

$$E_\phi = j \frac{\beta\eta}{r} \left[\frac{\sin(\beta r)}{\beta r} - \cos(\beta r) \right] \cos \theta \sin(\phi + \phi_0) \quad (11b)$$

where ϕ_0 is a constant to be determined. Based on E_θ and E_ϕ given above, the magnetic field in a magnetized ferrite sphere is determined by Faraday's law in tensor form; i.e.,

$$\vec{H} = j \frac{1}{\omega} [\mu_s]^{-1} \nabla \times \vec{E}. \quad (12)$$

For easy reference, $[\mu_s]^{-1}$ is given in (13), shown at the bottom of the page.

The electric field in the magnetized ferrite is then recomputed by Ampere's law

$$\vec{E} = -j \frac{1}{\omega \epsilon_r \epsilon_0} \nabla \times \vec{H}. \quad (14)$$

As expected, the electric field recomputed by (14) will differ from its approximation given in (11) for a given ϕ_0 . Within a frequency range of interest, the root-mean-square error can be computed. The aim is to search for an optimal ϕ_0 that yields a minimum error. It is found that both (12) and (14) can be approximately satisfied by choosing $\phi_0 = 120^\circ$. This value will be used for subsequent calculations.

VI. PERFORMANCE OF THE WAVEGUIDE Y-JUNCTION CIRCULATOR

For a circulator, $E_\theta H_\phi^* + E_\phi H_\theta^*$ evaluated at $r = R, \theta = \pi/2$ and $\phi = 0$ represents the power flow into a ferrite sphere at its equator. Note that for $E_\theta H_\phi^*(\phi = 2\pi/3)$ is vanishingly small compared with $E_\theta H_\phi^*$; therefore the former is ignored. While $E_\theta H_\phi^*(\phi = 2\pi/3)$ represents the power flows from port 1 to port 2, $E_\theta H_\phi^*(\phi = 4\pi/3)$ stands for the power leaked to port 3. Thus, the insertion loss τ and the isolation α of the circulator can be obtained by

$$(\text{dB}) = 10 \log \left[\frac{E_\theta H_\phi^*(\phi = 2\pi/3)}{E_\theta H_\phi^*(\phi = 0)} \right] \quad (15)$$

$$(\text{dB}) = 10 \log \left[\frac{E_\theta H_\phi^*(\phi = 4\pi/3)}{E_\theta H_\phi^*(\phi = 0)} \right]. \quad (16)$$

VII. ANALYTICAL AND EXPERIMENTAL RESULTS

Variation of the center frequency f_0 given in (9) is plotted in Fig. 2 for various R . The bias external magnetic field is 200 Oe. By symmetry the demagnetization factors of a sphere are equal; i.e., $N_x = N_y = N_z = 1/3$. Thus, $H_0 = 68.7$ Oe. Also included in Fig. 2 are the experimentally determined f_0 for two ferrite spheres of radii 0.5125 and 0.5625 mm. Good agreement between the analytical result and measurement is observed. This finding is consistent with

$$[\mu_s]^{-1} \mu_0 (\mu^2 - \kappa^2) = \begin{bmatrix} (\mu^2 - \kappa^2) \cos^2 \theta + \mu \sin^2 \theta & (\kappa^2 - \mu^2 + \mu) \sin \theta \cos \theta & -j\kappa \sin \theta \\ (\kappa^2 - \mu^2 + \mu) \sin \theta \cos \theta & (\mu^2 - \kappa^2) \sin^2 \theta + \mu \cos^2 \theta & -j\kappa \cos \theta \\ j\kappa \sin \theta & j\kappa \cos \theta & \mu \end{bmatrix} \quad (13)$$

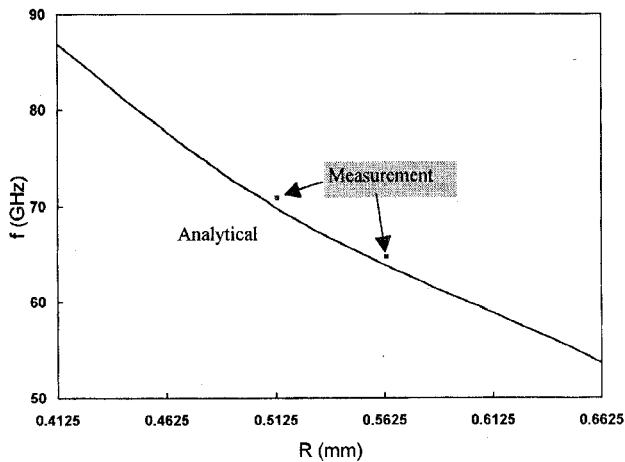


Fig. 2. Operating frequency of the circulator via the radius of the ferrite sphere.

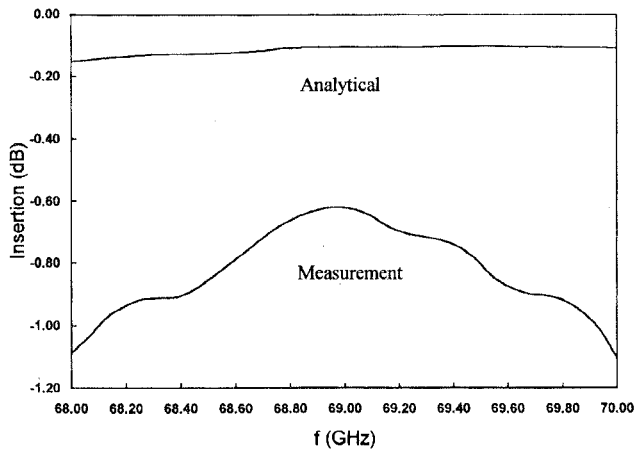


Fig. 3. Insertion loss of the circulator with a ferrite sphere.

that observed by Owen. One may conclude that the center frequency of a junction circulator with a ferrite specimen depends solely on the shape of the ferrite sample and is independent of the bias magnetic field.

Variation of the insertion loss τ is plotted in Fig. 3 for $68 \text{ GHz} < f < 70 \text{ GHz}$. Experimental results are also included in Fig. 3 for comparison. Within the frequency band of interest, the insertion loss is very small, less than 0.6 dB. A deviation of over 0.5 dB between the analytical prediction and the measurement is observed. This is expected because neither the reflection at the interface between the waveguide and the triangular junction, nor the reflection due to the impedance-matching patch, nor the reflection on the surface of the ferrite sphere has been considered in calculating τ . Variation of the isolation α with respect to f is sketched in Fig. 4. The shape of the analytical curve is very close to that of the experimental plot. On the average, the measured insertion loss is twice as high as that predicted by formula. This discrepancy is probably due to the omission of energy losses inside the waveguide junction, the matching plate, the epoxy glue, and the ferrite sphere. Nevertheless, as the first attempt to derive a simple method to analyze this circulator, the observed agreement should be accepted as satisfactory.

Should the center frequency of a circulator f_0 be defined as the minimum in the α plot, the theoretically determined f_0 is 69.58 GHz. It is identical to the value predicted by (9). Experimentally, $f_0 = 69.52 \text{ GHz}$, which represents a shift of only 0.09%.

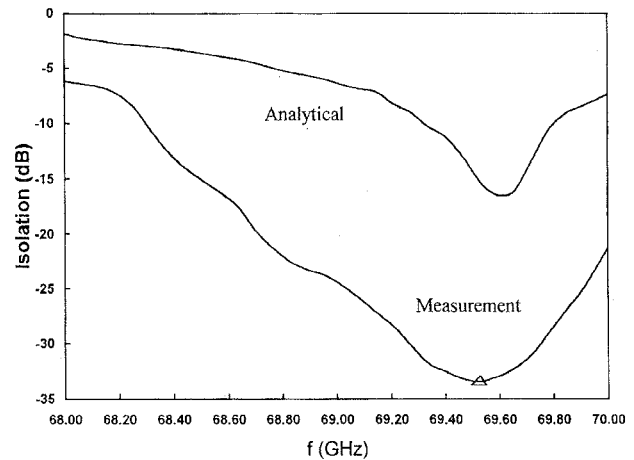


Fig. 4. Isolation of the circulator with a ferrite sphere.

VIII. CONCLUSION

A novel H-plane waveguide Y-junction circulator with a ferrite sphere is designed for uses in the 4-mm-wave range. A tiny ferrite sphere is used to replace a ferrite cylinder because sphere can be easily manufactured as if it were a ball bearing. With an insertion loss of less than 1 dB and an isolation of more than 20 dB in the frequency range of 68.7–70.1 GHz, the performance of this circulator is comparable to the conventional circulator with a cylindrical ferrite. The major gain is the lower production cost.

A simple formula is derived to determine the center frequency of a magnetized ferrite sphere. Excellent agreement between theoretical and experimental results is observed. Instead of solving the complex tensor equations, a fast method is developed to evaluate various properties of a circulator with a ferrite sphere. Satisfactory results are obtained. The accuracy of the approximation method introduced in this study can be enhanced by taking all energy losses and wave reflections into consideration. On the other hand, one may try to use more eigenmodes. For the latter modification, however, a more rigorous optimization method is needed to find the coefficients of expansion.

It must be emphasized that all formulas developed in this study are given in a closed form. All results can be enumerated in seconds using a PC. Finally, but not the least, more applications of ferrite spheres in submillimeter waves or optoelectronics are envisaged because making a tiny perfect sphere is easier than producing a larger one.

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